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Time-Reversibility, Instability and Thermodynamics in N -body Systems interacting with Long-Range Potentials

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Chaos is considered to be one of the origins of irreversibility appearing in macroscopic systems, such as N -body systems [1]. Accordingly, stability of the system has been extensively examined numerically, especially in an N -body system interacting with short-range potentials. However, numerical irreversibility due to round-off errors may behave as if it were a physical irreversibility, although it is not a physical one [2]. The influence of round-off errors should be a more serious problem in an N -body system interacting with 'long-range potentials', e.g., self-gravitating systems [3]. For instance, in a typical star-rich cluster with a million stars, each star feels enough of the granularity of the gravitational field of the other stars that the consequent perturbations lead to a total loss of memory of the initial conditions of its orbit [4]. However, in N -body simulations, numerical fluctuations due to round-off errors could behave as if they were the physical perturbations. Therefore, we have investigated numerical irreversibility and instability of the self-gravitating system, through the Loschmidt reversibility paradox based on a velocity inversion technique [5]. Consequently, the memory loss time, when the simulated trajectory completely forgets its initial conditions, increases approximately linearly with the Lyapunov time.

Because of long-range attractive potentials, such a self-gravitating system exhibits several peculiar features, such as gravothermal catastrophe, negative specific heat and nonextensive statistical mechanics [6, 7]. In particular, the negative specific heat causes thermodynamic instability during dynamical evolutions of the system and, therefore, it has been investigated theoretically and numerically from a thermodynamic viewpoint [8, 9]. However, velocity distributions and velocity relaxations have not yet been extensively discussed in long-term nonequilibrium processes except for a few studies [10, 11], although they should play an important role in the thermodynamic properties and irreversibility of the system [12]. For example, Iguchi *et al.* proposed universal non-Gaussian velocity distributions for a spherical collapse in a violent gravitational process of a collisionless stage ($t < \tau_r$) [10], while Ispolatov *et al.* discussed Gaussian velocity distributions in core-halo states in a collisional stage ($t \gg \tau_r$) [11]. (Here τ_r represents the relaxation time, which is driven by the two-body encounter [6]). These works may suggest that long-range attractive interacting systems should finally relax towards a Boltzmann-like state thorough a collapse process [12].

In the present study, to clarify irreversibility and thermodynamic properties of self-gravitating N -body systems, we numerically examine long-term evolution of those systems, from an early relaxation to a collapse, especially focusing on velocity relaxations [12]. For this purpose, we observe a cold collapse process under a restriction of constant mass and energy. To simulate the dynamical evolution, we consider a typical small N -body system ($N = 125$) enclosed in a spherical container with adiabatic walls [11, 12]. In our units, the relaxation time τ_r of the system is evaluated as $\tau_r \approx 0.5$ [2]. To examine whether the simulated velocity distribution function is Gaussian, a q -Gaussian distribution function is defined as $f_q(v) = A[1 - B(1 - q)v^2]^{1/(1-q)}$, where q is the Tsallis entropic parameter [7]. (A and B correspond to a normalization parameter and an inverse of temperature, respectively.) Moreover, to examine an overview of the velocity distributions, we employ the normalized ratio of velocity moments VM, i.e., a ratio of velocity moments $\langle v_i^2 \rangle^2 / \langle v_i^4 \rangle$ is normalized by a specific value corresponding to a Gaussian distribution [12]. (Here v_i and $\langle X \rangle$ represent the velocity speed of the i -th particle and the mean of X at time t , respectively.) When the velocity or speed distribution is Gaussian, q or VM approaches 1, respectively.

In this abstract, we observe time evolutions of typical properties of the cold collapse process. As shown in Fig. 1, VM and q deviate from 1 in an early relaxation ($t < 1$). Accordingly, the velocity distribution is non-Gaussian (VM < 1 and $q > 1$) in this stage. Thereafter, VM and q further deviate

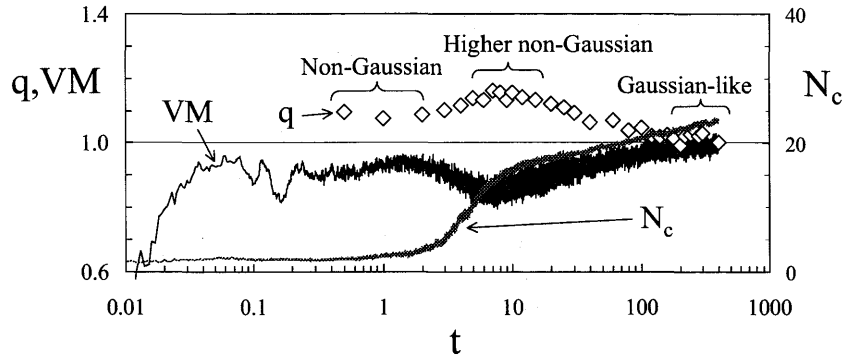


Figure 1: Time evolutions of the properties of the cold collapse process [12]. To mimic a cold collapse, the initial kinetic energy is set to be negligible values smaller than the order of 1% of the total energy. The total energy is $\varepsilon = -0.8$. The results are averaged over at least 30 simulations. To determine q , the q -Gaussian distribution function $f_q(v)$ is fitted with the simulated velocity distribution function.

from 1. In fact, the number N_c of core particles starts to increase at $t \approx 1$ –2. This suggests that the velocity distribution undergoes higher non-Gaussian distributions, especially when the core forms rapidly in the collapse process. However, the velocity distribution gradually relaxes toward a Gaussian-like distribution (VM, $q \sim 1$), after the core forms sufficiently ($t \gtrsim 200$). We found that the velocity distribution does not monotonically relax towards a Gaussian-like distribution. We clearly show such a transition of the velocity distribution, based not only on the Tsallis entropic parameter q but also on the normalized ratio of velocity moments VM. Of course, we have not yet clarified irreversibility in the strong nonequilibrium process of self-gravitating systems. However, our studies open up a new theoretical and numerical approach for examining irreversibility, instability and thermodynamics appearing in long-range interacting systems.

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